# Job Specialization and Labor Market Turnover\*

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#### Abstract

This paper studies the decline in labor market turnover over recent decades, in particular, the job finding and separation rates. I analyze the role of an increase in specialization of jobs in accounting for this decline, where specialization is defined as the impact of mismatch on match productivity. Combining individual level data from NLSY79 and NLSY97 with data on skills from ASVAB and O\*NET, I empirically estimate job specialization and show that the specialization has increased over time. To quantify the impact of this increasing specialization on labor market turnover, I build an equilibrium search and matching model with two-sided ex-ante heterogeneity and endogenous separations. The calibrated model shows that higher job specialization leads to a decline in both job finding and separation rates. As specialization increases, firms and workers become more selective in forming matches. Thus, well-matched firms and workers choose to remain in their matches longer, while bad matches get destroyed faster. Since higher specialization leads to an increase in the proportion of good matches in the economy, it results in a decline in the labor market turnover.

JEL codes: E24, J63, J64. Keywords: Turnover, Specialization, Mismatch, Sorting.

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# 1. Introduction

Various measures of labor market turnover such as job finding and separation rates obtained from worker flow data, or job creation and destruction rates obtained from firm data, have exhibited a secular decline over the past four decades. Figure 1 shows the evolution of separation and job finding rates constructed by Shimer (2012) using the CPS microdata. While the monthly separation rate averaged around 4% during the 1980s, it has declined to around 2% in recent years. The job finding rate also shows a decline over time, from around 44% on average before 1995 to around 30% in the past decade. I focus on investigating and explaining these observed falls in job finding and separation rates.

Even though there is a growing empirical literature documenting a secular decline in labor market turnover, there is still no consensus on the underlying economic factors driving it.<sup>1</sup> The fall in labor market turnover could be due to an increase in the costs of making labor market transitions. On the other hand, labor market turnover could be declining because there is reduced need for making such transitions. We must identify the main forces generating reduced labor market turnover if we are to understand its consequences for the aggregate economy now and in the future. I propose an explanation based on measured increases in job specialization and evaluate its effect on turnover using a calibrated equilibrium search model.

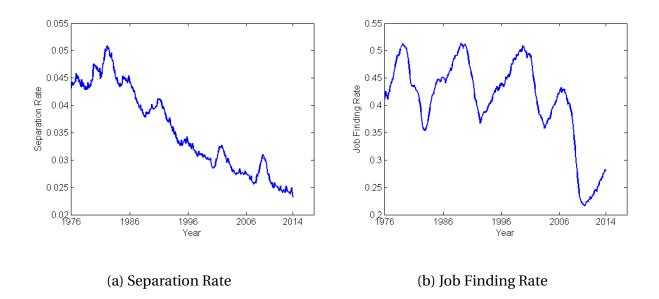
I argue that there has been an increase in the specialization of jobs, and that this has been an important factor explaining the fall in labor market turnover that we see in the data. Job specialization is defined as the impact of mismatch on match productivity, where mismatch is the distance between the skills or ability of a worker and the skill requirements of their job. If a job has zero specialization, then any worker with any skill level is suitable for the job, and so, mismatch has zero effect on match productivity. On the other hand, if a job is highly specialized, even a small amount of mismatch can have a large negative impact on match productivity. As job specialization increases, firms become more reluctant to enter into matches with workers ill-suited for the specific skill needs of their jobs. This leads to more skill-compatible matches and reduced labor market turnover, as explained below.

<sup>&</sup>lt;sup>1</sup>Some of the papers documenting this decline are Davis (2008), Hyatt and Spletzer (2013), Davis and Haltiwanger (2014), Decker et al. (2014), and Molloy et al. (2016).

A reasonable estimate of specialization requires an empirical measure of mismatch across the jobs in the economy. I follow the framework of Guvenen et al. (2020) in constructing such a measure. The data on individual workers are obtained from 1979 and 1997 cohorts of the National Longitudinal Survey of Youth, namely the NLSY79 and the NLSY97. Since NLSY follows the same cohort of individuals over time, I make use of both NLSY79 and NLSY97 samples to examine how job specialization has changed over time. In particular, I use data from NLSY79 for the years 1978–1995, and data from NLSY97 for 1996–2014 to capture the change in specialization over time.

Sample members of both NLSY79 and NLSY97 undertake an occupational placement test called Armed Services Vocational Aptitude Battery (ASVAB). The test scores provide detailed measures of each individual's skills along various dimensions. NLSY also contains various measures documenting the social skills of a worker. I aggregate selected test scores to construct a skill measure reflecting the verbal, math, and social skills of each worker across both the cohorts. The data on the skill requirements of jobs is taken from the Occupational Information Network (O\*NET). This database provides detailed requirements along a large number of skill dimensions for various occupations. As with worker skills, I combine data on multiple skills dimensions to obtain an aggregated measure of verbal, math, and social skill requirements for each occupation. Having obtained the skill endowments of workers and the skill requirements of their jobs, I calculate mismatch as the distance between a worker's skill endowments and their job's skill requirements. Finally, I estimate job specialization for both NLSY79 and NLSY97 individuals by using a standard Mincerian wage regression augmented with the empirical measure of mismatch.

The regression estimates from both NLSY79 and NLSY97 show that jobs on average are specialized, i.e. mismatch has a negative impact on productivity. Over the years 1978–1995, moving from the best match (lowest mismatch) to the worst match (highest mismatch) is associated with 15% decline in wages. Comparing this with the corresponding estimate from NLSY97, I find that the specialization has increased over 1996–2014, with the wage loss associated with mismatch increasing to 25%. I propose that this 10 p.p. increase in the cost of mismatch may have a significant role in explaining the decline in labor market turnover.



*Note:* Separation and job finding rates from 1976 to 2014. Data constructed by Robert Shimer using CPS micro data. For additional details, please see Shimer (2012).

Figure 1: Labor Market Turnover

Further, I construct the distribution of employment over mismatch and analyze how this distribution has changed over time. Consistent with mismatch being costly, the share of employment is a decreasing function of mismatch, with majority of matches formed with low mismatch. More importantly, comparing the average distribution over the period of 1978–1995 (NLSY79) with that of 1996–2014 (NLSY97), I find that the employment distribution has moved towards lower mismatch. Workers and firms sort themselves into better matches on average in 2000s compared to the earlier period.

To quantitatively study how increases in job specialization impact labor market turnover, I develop an equilibrium labor search and matching model with two-sided ex-ante heterogeneity. Individual workers are assumed to differ in their skill endowments, while jobs have differing skill requirements, and they are located on a unit circle according to their skills. The productivity of a match decreases with mismatch, which is defined as the distance between the worker (skills) and the firm (skill requirements) in a match. And, analogous to the empirical specification, job special-

ization in the model is defined as the extent to which any given level of mismatch reduces the productivity of the match. Calibrating the model to the US, I find that the increase in job specialization can lead to a decline in both separation and job finding rates, as observed in the data.

As jobs become more specialized, workers and firms grow more selective about which matches they enter into. This has two opposing effects on labor market turnover. First, well-matched firms and workers choose to remain in their matches longer, as they know it is more difficult to find an acceptable match in the future, resulting in lower separation rates. On the other hand, since increased specialization raises the cost of mismatch, ill-suited firms and workers choose to abandon their matches more quickly, leading to higher separation rates. To disentangle the effect that has a larger impact on aggregate turnover measures, I examine changes in the distribution of employment over mismatch. Since an increase in specialization raises the cost of mismatch, more firms and workers choose to move toward better matches. With increased sorting, a majority of employment faces lower separation rates while a minority faces higher separation rates. This causes a fall in the aggregate separation rate. Further, increased selectivity in match formation reduces the incentive for the firms to post vacancies. This reduces the labor market tightness which in turn causes a decline in the job finding rate.

I also find that changes in job specialization has implications for wage dispersion and aggregate labor productivity. Increase in specialization generates higher dispersion in wages through increased sorting in the labor market. This is consistent with the findings of Barth et al. (2016) and Song et al. (2019), who attribute higher sorting to be a major contributor of increase in wage inequality in the US. Higher job specialization also has an adverse effect on aggregate labor productivity. Although increased job specialization causes workers and firms to move to better matches, the resulting fall in mismatch does not fully compensate for the increased productivity costs of mismatch. This causes a fall in aggregate labor productivity in my steady state analysis, which is in line with the findings of Byrne et al. (2016), who document a slowdown in the growth rate of labor productivity after 2000.

This paper contributes to a growing literature engaged in understanding the sources behind the secular decline in labor market turnover. Davis et al. (2010) argue that a

fall in the job destruction rate can lead to a decline in unemployment inflows and show that the observed decline in the job destruction rate can account for 28% of the decline in unemployment inflows from 1978 to 2005. Cairo (2013) argues that increased training requirements for jobs may explain the fall in aggregate job flows. She models training costs as a fixed cost and shows that increases in these costs makes firms more reluctant to adjust their employment, resulting in reduced job flows. Fujita (2018) proposes increased turbulence as a factor driving this fall in turnover. A rise in turbulence is modeled as an increased risk of skills becoming obsolete during unemployment. As this risk increases, workers are less willing to leave their jobs, leading to a decline in the aggregate separation rate.

This paper also adds to the literature of equilibrium search models with heterogeneous workers and firms. I extend the models of Marimon and Zilibotti (1999) and Gautier et al. (2010) by incorporating endogenous separations to study changes in labor market turnover. Finally, this paper contributes to the literature on wage dispersion by demonstrating that higher job specialization can help explain both the rise in sorting and increase in wage dispersion observed in the data.

The rest of the paper is organized as follows. Section 2 presents the empirical framework and estimates of job specialization. Section 3 describes the model and its equilibrium conditions. Section 4 presents the calibration strategy while the main results of the paper are presented and analyzed in section 5. Section 6 concludes. Data description, supplementary empirical evidence and proofs are provided in the appendix.

# 2. Empirical Evidence

In this section, I show that, job specialization as measured by the cost of mismatch has increased over time. In order to accomplish this, I construct an empirical measure of mismatch and estimate specialization across two cohorts of individuals, namely NLSY79 and NLSY97. Finally, I also document that the distribution of employment has shifted towards lower mismatch over time.

## 2.1 Data

The labor market information for individual workers are obtained from National Longitudinal Survey. NLSY follows a nationally representative sample of individuals across years and it consists of two cohorts, NLSY79 and NLSY97. NLSY79 contains yearly data on individuals who were between the ages of 14 and 22 years on January 1, 1979, while NLSY97 contains data on individuals who were 12 to 16 years old as of December 31, 1996. Since my main interest is to estimate the changes in job specialization over time, I make use of both NLSY79 and NLSY97 to capture the time change. More precisely, I use data from NLSY79 for the years 1978 to 1995 while the data for 1996 to 2014 are obtained from NLSY97. In both the datasets, I consider only the crosssectional sample and do not include either the supplemental or the military sample. I also consider only individuals who entered the primary labor market after being selected into the sample. I also restrict my analysis to individuals aged between 16 and 35 years to make the sample comparable across NLSY79 and NLSY97 cohorts. Finally, my NLSY79 sample covers the years 1978 to 1995 with 44,886 individual-year observations, while the NLSY97 sample runs from 1996 to 2014 with 41,864 individual-year observations. The details of the sample selection are given in appendix A.

### Worker's Skills

To estimate individual level mismatch, I first need to obtain data on the skills of the workers across three dimensions — math, verbal, and social. One of the reasons to use NLSY data is that, the respondents in both NLSY79 and NLSY97 were administered an occupational placement test called Armed Services Vocational Aptitude Battery (ASVAB). This test, administered by the U.S. Department of Defense, gives detailed information on worker's skills across multiple dimensions. ASVAB test administered to NLSY individuals had 10 components.<sup>2</sup> Following Guvenen et al. (2020), I use the test scores in Arithmetic Reasoning and Mathematics Knowledge to obtain the *math* skill of the individuals while the *verbal* skill is estimated using Word Knowledge and Paragraph Comprehension. Since the respondents were of different ages

<sup>&</sup>lt;sup>2</sup>The 10 components are arithmetic reasoning, mathematics knowledge, paragraph comprehension, word knowledge, general science, numerical operations, coding speed, automotive and shop information, mechanical comprehension, and electronics information

when the test was administered, and since age can have a systematic impact on the test scores, I normalize these test scores using age-specific means and variances.

Since our objective is to compare the estimates of specialization from NLSY79 with those of NLSY97, it is important that the measure of worker skills is consistent across the two cohorts. Even though the sample members of both NLSY79 and NLSY97 were administered the ASVAB test with the same 10 components, the format of the test was different. The ASVAB administered to NLSY79 members was a paper and pencil test (P&P) while the format of NLSY97 was computer adaptive test (CAT). Owing to this, ASVAB test scores are not directly comparable across NLSY79 and NLSY97. To address this issue, Segall (1997) provides a mapping between the two test scores. Segall (1997) randomly assigned individuals to both P&P and CAT, and estimated equipercentile mappings of the test scores. Following Altonji et al. (2012), I use this mapping to convert the CAT based NLSY97 scores to its equivalent P&P scores for comparability with NLSY79.<sup>3</sup>

In addition to verbal and math skills obtained from ASVAB, NLSY also contains data on social abilities of the individuals. I follow Deming (2017) to construct social skills that maximize comparability across both NLSY79 and NLSY97 waves. I use 2 variables — self-reported sociability, and self-reported sociability at the age of 6 years (retrospective) to estimate social skills for the NLSY79 cohort. For the NLSY97, I combine the two questions capturing the extroversion factor of the individuals to estimate their social skills.<sup>4</sup> Extroversion is one of the factors of Big 5 personality inventory as detailed in Goldberg (1993). As with ASVAB scores, the social skills are normalized using age-specific means and variances to remove the impact of age on these measures.

### Job's Skill Requirements

The data on skill requirements of different occupations are obtained from the O\*NET database. This database put together by the US Department of Labor gives information on knowledge, skills, and abilities required to perform around 974 different

<sup>&</sup>lt;sup>3</sup>I thank Fabian Lange for providing this mapping for NLSY97 scores.

<sup>&</sup>lt;sup>4</sup>In particular, the variables I use are Personality Scale: Extraverted, Enthusiastic (YTEL-TIPIA 000001) and Personality Scale: Reserved, Quiet (YTEL-TIPIA 000006).

occupations. For each of these occupations, this database provides a score of importance of 277 different descriptors. Following Guvenen et al. (2020), I choose 26 descriptors to calculate skill requirements across math and verbal dimensions. I also choose six descriptors to capture the social skill requirements across different occupations.<sup>5</sup> Finally, I match the O\*NET occupation codes with the corresponding census codes of NLSY79 and NLSY97 using the crosswalks developed by Autor and Dorn (2013). Appendix A provides more details on this.

### **Skill Dimensions**

I now match the verbal and math skill endowments (ASVAB) of the individuals with the corresponding skill requirements (O\*NET) of their occupations. To achieve this, we need to map the 26 categories of O\*NET to the four ASVAB test components that were chosen earlier. For this purpose, I make use of the crosswalk put together by the Defense Manpower Data Center (DMDC). The DMDC provides a relatedness score for each of the 26 O\*NET descriptors for mapping onto the ASVAB test categories. For each ASVAB test component, we can create an equivalent O\*NET requirement by calculating the weighted sum of the 26 descriptors, where the weights are given by the relatedness score. At the end of this process, we obtain four O\*NET measures that can be compared with the scores of four ASVAB test categories — Word Knowledge, Paragraph Comprehension, Arithmetic Reasoning, and Mathematics Knowledge.

After normalizing each of ASVAB test categories to have a unit standard deviation, I combine the four test components into two skill dimensions, namely verbal and math, using Principal Component Analysis (PCA). The verbal score is the first principal component of Word Knowledge and Paragraph Comprehension, while the math score is the first principal component of Arithmetic Reasoning and Mathematics Knowledge. I repeat this procedure for O\*NET measures to generate the skill requirements along verbal and math dimensions. Following Lise and Postel-Vinay (2020), I rescale the math and verbal skill endowments and their corresponding skill requirements to lie in [0, 0.5].<sup>6</sup>

<sup>&</sup>lt;sup>5</sup>Table C.3 in Guvenen et al. (2020) provides the list of descriptors used to construct these measures of skill requirements.

<sup>&</sup>lt;sup>6</sup>Lise and Postel-Vinay (2020) uses linear transform to represent their skill measures in [0, 1]. I normalize the skills so that they lie in [0, 0.5] to be consistent with my model framework.

Moving on to the social dimension, I collapse the six descriptors from O\*NET, after standardizing each score to have a standard deviation of one, into a single social requirement by taking the first principal component. Similarly, on the worker's side, the standardized measures of extroversion are combined to obtain the social skill endowment. Just like in math and verbal dimensions, social skills and requirements are rescaled to be in the interval of 0 and 0.5. We are now able to characterize each worker using their math, verbal, and social skills and their occupations using its math, verbal, and social skill requirements.

## 2.2 Mismatch

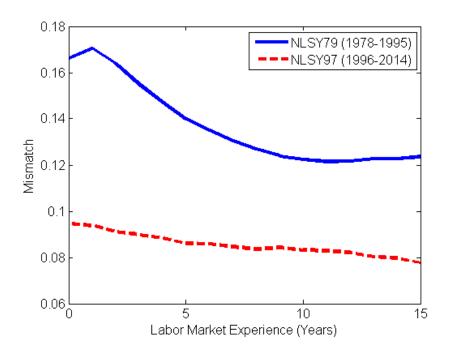
I now bring together the measures on skill endowments and skill requirements that we constructed to obtain an estimate of mismatch. Mismatch  $x_{i,c}$  of worker *i* in occupation *c* is given by the distance measure

$$x_{i,c} = \sum_{j=1}^{n} \left[ \omega_j \times |\hat{a}_{i,j} - \hat{r}_{c,j}| \right],\tag{1}$$

where  $\hat{a}_{i,j}$  is the estimated skill endowment of worker *i* in dimension *j*,  $\hat{r}_{c,j}$  is the estimated skill requirement of occupation *c* in dimension *j*, and *n* is the total number of skill dimensions (here 3). The weight,  $\omega_j$ , gives the relative importance of dimension *j* to the overall mismatch. Following Guvenen et al. (2020), I use the factor loadings of the first principal component normalized to sum to 1 as weights. Thus, mismatch is defined as the distance between a worker's skill endowment and their job's skill requirement across all the skill dimensions.

Figure 2 plots the average mismatch over labor market experience for both NLSY79 and NLSY97 workers. Mismatch on average declines with labor market experience, implying that workers move to better matches over their lifetime.<sup>7</sup> More importantly, I find that mismatch on average is lower among NLSY97 individuals compared to NLSY79 cohort across different years of experience. On average, workers present in the labor market during 1996–2014 (NLSY97) got matched to jobs with higher match

<sup>&</sup>lt;sup>7</sup>Workers even after taking their ASVAB tests do not choose their ideal jobs immediately because NLSY respondents were not told their exact scores but only given a range in which their score lies. Also, the worker's decision to take up a job might have been influenced by other factors on top of their ASVAB scores. Refer to Guvenen et al. (2020) for a detailed discussion on this.



*Note:* This figure plots the average mismatch over labor market experience of workers for both NLSY79 and NLSY97. Individual level mismatch is calculated using equation (1).

Figure 2: Average Mismatch over Labor Market Experience

quality compared to those in 1978–1995 (NLSY79). This is consistent with my hypothesis, with increase in the cost of mismatch, workers and firms sort themselves better in the labor market, which in turn could lead to a decline in job finding and separation rates. Next, I estimate job specialization for both NLSY79 and NLSY97 cohorts and show that the cost of mismatch has indeed increased over time.

# 2.3 Job Specialization

The specialization of a job measures the impact of mismatch on the match productivity. For a job with zero specialization, mismatch will have zero impact on the productivity. This means any worker with any skill set will be able to perform that job equally well. On the other hand, if a job is highly specialized, mismatch can have a large negative impact on the productivity of the match. Thus, we can empirically estimate job specialization by regressing match productivity on our measure of mismatch. Since match productivity is not directly observable, I use hourly real wages

(indexed to 2009 dollars) to capture the match quality of the NLSY workers. I use an empirical specification that closely follows Guvenen et al. (2020) to estimate job specialization for both NLSY79 and NLSY97 cohorts. The log wage  $w_{i,j,c,t}$  of individual *i* in job *j* belonging to occupation *c* at time *t* is given by

$$lnw_{i,j,c,t} = \phi x_{i,c} + \alpha_1 \bar{a}_i + \alpha_2 \bar{r}_c + \alpha_3 \Big( \bar{a}_i \times T_{i,c,t} \Big) + \alpha_4 \Big( \bar{r}_c \times T_{i,c,t} \Big) + \alpha_5 O J_{i,t} + \Psi(T_{i,c,t}) + \Psi(J_{i,j,t}) + \Psi(E_{i,t}) + Z'_{i,t} \chi + \epsilon_{i,j,c,t}.$$
(2)

This is a standard Mincerian wage regression augmented with the measure of mismatch,  $x_{i,c}$ , suffered by individual *i* working in occupation *c*. The average skills of the worker  $(\bar{a}_i)$  and the average skill requirements of the occupation  $(\bar{r}_c)$  are introduced to capture the individual fixed effects. I also include their interactions with occupational tenure  $(T_{i,c,t})$ . The regression equation also controls for job tenure  $(J_{i,j,t})$ , occupational tenure  $(T_{i,c,t})$ , and labor market experience  $(E_{i,t})$  using a cubic polynomial  $\Psi(.)$ . While estimating this regression, I also include a dummy variable,  $OJ_{i,t}$ denoting a continuing job, a vector of education and demographic characteristics  $Z_{i,t}$ , and industry and occupation fixed effects. The coefficient  $\phi$  is our estimate of job specialization as it measures the impact of mismatch on wages. By estimating this regression for both NLSY79 and NLSY97 workers, and comparing the coefficient  $\phi$ , we can capture the changes in job specialization, if any.

In order to estimate how the specialization has changed over time, it is necessary to use both NLSY79 and NLSY97 samples. This is because NLSY follows the same cohort of individuals over time. Thus, as we move ahead in time, the average labor market experience of our sample also increases. As workers spend more time in the labor market, they might learn more about their skills or learn about various job opportunities, and hence move to a job which is the closest match to their skills.<sup>8</sup> Thus, an aging sample of workers can mechanically lead to lower mismatch on average, and hence bias our estimates of specialization.<sup>9</sup> Any changes in specialization attributed

<sup>&</sup>lt;sup>8</sup>The literature of learning models deals with workers who learn about their own skills, the match quality or other attributes of jobs over the time of experience. Some of the papers that explore these topics include Jovanovic (1979), Sanders (2014), and others.

<sup>&</sup>lt;sup>9</sup>In our sample, the average mismatch decreases with the labor market experience as seen from figure 2.

to this can be called as the *cohort effect*. Thus, by using two different cohorts of individuals that are comparable, namely NLSY79 and NLSY97, we can filter out the *cohort effects*, and extract the actual *time effects* of the changes in specialization.

Before we proceed with estimating our wage equation (2), we need to address two issues that could potentially bias our results. The first issue is the presence of serial correlation in the residuals. I take care of this by clustering the standard errors at the individual level. Alternatively, I also follow Guvenen et al. (2020) in introducing an AR(1) structure for the errors and estimating the equation using GLS. As I will show in the next section, both the estimation strategies yield similar results.

The second issue is the endogeneity of tenure variables in the regression equation (2), as both occupational tenure and wages are a function of the underlying match quality. As proposed by Altonji and Shakotko (1987), I instrument both job tenure and occupational tenure variables with the deviations from their means for a given job match. Labor market experience, continuing job dummy and the interaction terms involving occupational tenure are also instrumented using the same procedure. The estimates of job specialization are similar with and without instrumental variables, as we shall see next.

### Baseline

Table 1 shows the main results of the regression analysis when the standard errors are clustered at the individual level. The first column lists the regression coefficients of NLSY79 sample using OLS, while the second column IV-OLS shows the OLS estimates when the tenure and experience variables are instrumented to take care of the endogeneity. The next two columns shows the estimates of OLS and IV-OLS for NLSY97 cohort. Across all specifications, mismatch has a negative and significant effect on the real wages, i.e. jobs on average are specialized. During the period of 1978 to 1995, mismatch has an estimated coefficient of -0.3190 under IV-OLS. This can be interpreted as, moving from the best match (mismatch of zero) to the worst match (mismatch of 0.5) is associated with 15% fall in wages.<sup>10</sup>

More importantly, the estimates in table 1 show that the jobs have become more specialized after 1995. During the period from 1996 to 2014, the wage loss associated

<sup>&</sup>lt;sup>10</sup>The effect of mismatch on wages is obtained as  $\exp(-0.319 \times 0.5) - \exp(-0.319 \times 0) = -0.1474$ .

	NLSY79 (1	978–1995)	NLSY97 (1996–2014)		
	(1)	(2)	(3)	(4)	
	OLS	IV-OLS	OLS	IV-OLS	
Mismatch	-0.3051***	-0.3190***	-0.5836***	-0.5768***	
	(0.0795)	(0.0818)	(0.0824)	(0.0866)	
Worker Skill (Average)	0.2963***	0.3035***	0.1225***	0.0713*	
	(0.0448)	(0.0493)	(0.0367)	(0.0411)	
Occ. Requirement (Average)	0.1846***	0.0642	0.2839***	0.2235***	
	(0.0477)	(0.0516)	(0.0442)	(0.0509)	
Skill $\times$ Occ. Tenure	0.0365***	0.0345***	0.0138	0.0342***	
	(0.0095)	(0.0088)	(0.0120)	(0.0091)	
Requirement $\times$ Occ. Tenure	0.0214***	0.0508***	0.0472***	0.0705***	
	(0.0077)	(0.0072)	(0.0114)	(0.0085)	
N	44886	44886	41864	41864	

### Table 1: Job Specialization in NLSY79 and NLSY97

*Note:* This table presents the estimates of the wage regression (2) for both NLSY79 and NLSY97. Each regression also includes job tenure, occupational tenure, experience, demographic characteristics, industry and occupation fixed effects. IV-OLS instruments for tenure and experience variables following Altonji and Shakotko (1987). Standard errors in parentheses are clustered at individual level. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01.

with the worst match is around 25% compared to 15% in the previous period, and this increase in specialization is statistically significant at 5%.<sup>11</sup> In line with our expectations, I also find that, workers with higher skills earn more, and occupations with higher skill requirements pay more on average. Alternatively, estimating the wage

 $^{11}$  To statistically test the change in specialization, I calculate the Z statistic

$$Z = \frac{\hat{\phi}_{79} - \hat{\phi}_{97}}{\sqrt{(s.e_{\cdot\phi,79})^2 + (s.e_{\cdot\phi,97})^2}}$$

where  $\hat{\phi}_{79}$  and  $\hat{\phi}_{97}$  are the coefficients of mismatch in NLSY79 and NLSY97, while  $s.e._{\phi,79}$  and  $s.e._{\phi,97}$  are their corresponding standard errors. Substituting the estimated coefficients and the standard errors, the *Z* statistic has a value of -2.16, and hence the increase in specialization is significant at 5%.

	NLSY79 (1	NLSY79 (1978–1995)		996–2014)	
	(1)	(2)	(3)	(4)	
	GLS	IV-GLS	GLS	IV-GLS	
Mismatch	-0.2294***	-0.2404***	-0.6561***	-0.6643***	
	(0.0482)	(0.0498)	(0.0502)	(0.0523)	
Worker Skill (Average)	0.3300***	0.3174***	0.2068***	0.1025**	
	(0.0339)	(0.0427)	(0.0342)	(0.0449)	
Occ. Requirement (Average)	0.1602***	0.0096	0.2718***	0.1863***	
	(0.0316)	(0.0363)	(0.0307)	(0.0368)	
Skill $\times$ Occ. Tenure	0.0256***	0.0267***	0.0003	0.0284**	
	(0.0053)	(0.0081)	(0.0072)	(0.0111)	
Requirement $\times$ Occ. Tenure	0.0206***	0.0478***	0.0324***	0.0502***	
	(0.0045)	(0.0069)	(0.0066)	(0.0104)	
N	35436	35436	32354	32354	

### Table 2: Job Specialization in NLSY79 and NLSY97

*Note:* This table presents the estimates of the wage regression (2) for both NLSY79 and NLSY97. Each regression also includes job tenure, occupational tenure, experience, demographic characteristics, industry and occupation fixed effects. IV-GLS instruments for tenure and experience variables following Altonji and Shakotko (1987). Standard errors in parentheses are estimated using GLS assuming AR(1) error structure. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01.

equation using GLS under the assumption of AR(1) residuals yield similar conclusions, as shown in table 2. The complete regression results are provided in appendix **B**.

## **Across Education**

I now look at the specialization of jobs held by workers with different educational attainment. Table 3 gives the estimates across different worker groups and how this specialization has evolved over time. During 1978–1995, workers who had not completed college perform jobs with very low specialization, while college graduates were associated with more specialized jobs. Using NLSY79 data, Guvenen et al. (2020) also

	NLSY79 (1	.978–1995)	NLSY97 (19	996–2014)	
	(1)	(2)	(3)	(4)	
	OLS	IV-OLS	OLS	IV-OLS	
Less than High School	-0.1522	-0.1301	-0.4592***	-0.4371***	
	(0.0961)	(0.1013)	(0.1020)	(0.1117)	
Some College	-0.1153	-0.2246	-0.5853***	-0.5572***	
	(0.1564)	(0.1615)	(0.1387)	(0.1452)	
More than College	-0.4534**	-0.4734**	-0.3389*	-0.4486**	
	(0.2281)	(0.2332)	(0.2003)	(0.2099)	

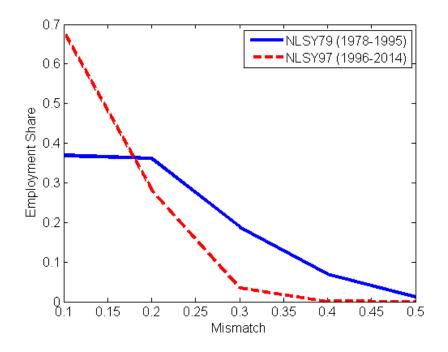
## Table 3: Job Specialization Across Education

*Note:* This table presents the estimates of job specialization from equation (2) across different educational attainment for both NLSY79 and NLSY97. Each regression also includes job tenure, occupational tenure, experience, demographic characteristics, industry and occupation fixed effects. IV-OLS instruments for tenure and experience variables following Altonji and Shakotko (1987). Standard errors in parentheses are clustered at individual level. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01.

shows that the cost of mismatch is higher for college graduates compared to others, in line with the findings documented here. Now, looking at the later period of 1996–2014, there is an increase in the specialization of jobs, particularly for workers with lower educational attainment. After 1995, specialization of jobs performed by workers who have not completed college has caught up with those done by college graduates.

# 2.4 Employment Distribution

The regression analysis showed us that the mismatch is costly and the cost of mismatch has increased over time. I now analyze how the employment is distributed over mismatch and the evolution of this distribution over time. Since we have a measure of mismatch for each employed individual across years, we can construct the distribution of employment each year by calculating the share of employment belonging to different values of mismatch. Figure 3 shows the average distribution



*Note:* This figure plots the average distribution of employment over mismatch for both NLSY79 and NLSY97. Individual level mismatch is calculated using equation (1).

Figure 3: Employment Distribution in NLSY79 and NLSY97

of employment over mismatch for the years 1978–1995 and 1996–2014, respectively. During both the periods, the measure of employment is decreasing with the level of mismatch. This is consistent with our earlier finding that mismatch is costly, and as a result, more matches are formed with lower levels of mismatch. More importantly, as the cost of mismatch increased over time, the distribution of employment has also moved toward lower mismatch region. During 1996–2014, almost 70% of the matches had mismatch less than or equal to 0.1, compared to around 40% in the previous period. Thus, workers and firms have formed better matches on average in 2000s compared to 1980s.

This finding is consistent with the literature on sorting in the labor market. Song et al. (2019) use administrative matched employer-employee data for the US to show that there has been an increase in sorting of high-wage workers with high-wage firms. Card et al. (2013) also document increased sorting based on worker and firm fixed effects for Germany, while Håkanson et al. (2020) find increase in sorting by cognitive

and non-cognitive skills for Sweden. Along these lines, I also show that sorting based on our measure of mismatch has increased over time, as more matches are formed with lower mismatch in the recent years compared to the earlier period.

## 2.5 Labor Market Turnover

		1978–1995	5	1996–2014			
	f	s	$\phi$	f	s	$\phi$	
Aggregate	0.4412	0.0416	-0.3190	0.3768	0.0284	-0.5768	
Less than High School	0.4471	0.0654	-0.1301	0.3839	0.0513	-0.4371	
Some College	0.4540	0.0330	-0.2246	0.3883	0.0254	-0.5572	
More than College	0.3712	0.0131	-0.4734	0.3315	0.0111	-0.4486	

Table 4: Labor Market Turnover

*Note:* This table shows the average labor market turnover and job specialization over the periods 1978–1995 and 1996–2014. *f* denotes the job finding rate and *s* the separation rate. Both the measures are constructed from monthly CPS microdata following Shimer (2012).  $\phi$  is the IV-OLS estimate of job specialization obtained from wage regression (2).

I now construct the measures of labor market turnover, namely the job finding rate and the separation rate using microdata from CPS. Job finding rate denotes the probability an unemployed worker becomes employed (also referred to as UE rate), while the separation rate captures the transition probability from employment to unemployment (EU rate). Following Shimer (2012), I use information on employment, unemployment and short-term unemployment (unemployment with duration less than 5 weeks) from CPS microdata to construct monthly job finding and separation rates. Figure 1 shows the evolution of labor market turnover from 1976 to 2014. As can be seen from the figure, there has been a secular decline in the separation rate from the early 1980s while job finding rate starts its decline after 1995. As shown in table 4, the aggregate separation rate averaged around 4.2% during 1978–1995 and has declined to around 2.8% after 1995. Similarly, prior to 1995, the average job finding rate was around 44% and this has reduced to around 38% over the recent period. Thus, both the measures of labor market turnover that I have considered in this paper

show a decline over time.

Next, I analyze this decline in turnover among workers of different educational attainment. Table 4 shows the measures of labor market turnover along with my estimates of job specialization across different education groups. Figures B1 and B2 in appendix B show the evolution of turnover across different educational attainment. Even though the turnover has reduced among all the education groups, both job finding and separation rates have declined much more for non-college workers compared to those who are college graduates. This change in turnover mirrors the change in job specialization that I documented in the previous section. Similar to the case of job finding and separation rates, the increase in specialization is primarily driven by the workers who have not completed their college education. This finding supports my hypothesis — increase in job specialization has led to a decline in labor market turnover. Importantly, the evidence presented here demonstrates a mere association between job specialization and labor market turnover. In order to establish causality, I write down a labor search model where changes in job specialization can endogenously affect labor market turnover.

# 3. The Model

This section presents a search and matching model with ex-ante heterogeneous workers and jobs distributed over a unit circle. I extend Marimon and Zilibotti (1999) and Gautier et al. (2006) by incorporating endogenous separations.

## 3.1 Environment

The economy consists of ex-ante heterogeneous workers and jobs. Workers having different skill endowments and jobs having different skill requirements are uniformly distributed over a circle of unit length. There is a unit measure of workers in total. At any given instant, a worker can be employed or unemployed. The model does not feature on-the-job search, and hence employed workers have to go through unemployment before changing jobs. Let the total measure of firms located on the circle be M. At each instant, the firm is either vacant or it is matched with a worker and

involved in production. Unmatched firms need to pay a cost to post vacancy. Finally, existing matches face idiosyncratic productivity shocks  $\epsilon \in [0, \overline{\epsilon}]$  that arrive at the rate  $\lambda$  from a distribution *F*.

## 3.2 Match Productivity

The productivity of a match is a decreasing function of the distance between the worker and the firm in a match. Let a worker be located at  $w \in [0, 2\pi]$  and a firm at  $f \in [0, 2\pi]$  on the circle.<sup>12</sup> Let  $\eta(\widehat{f, w})$  denote the productivity of this match, where  $\widehat{f, w} \in [0, \frac{1}{2}]$  is the arc-length (distance) between the worker and the firm. We can interpret  $\widehat{f, w}$  as the measure of mismatch in our model framework. In our quantitative exercise, the match productivity  $\eta(\widehat{f, w})$  takes the functional form

$$\eta(\widehat{f,w}) = 1 - \gamma \widehat{f,w}.$$
(3)

Just as in the empirical framework,  $\gamma$  measures the impact of mismatch on match productivity, i.e., job specialization. When  $\gamma$  takes the value of zero, mismatch has zero effect on productivity, and hence there is no job specialization. A higher value of  $\gamma$  signifies a larger (negative) impact of mismatch on productivity, and hence greater job specialization. Thus, this specification of match productivity is consistent with our empirical counterpart.

## 3.3 Labor Market Matching

Workers and firms are involved in random search, and hence, any unemployed worker can meet and be interviewed by any vacant firm on the circle. Let  $v : [0, 2\pi] \rightarrow \mathbb{R}^+$  denote the density of vacancies at location f, and  $u : [0, 2\pi] \rightarrow [0, 1]$  denote the density of unemployed at location w. The matching function  $m : \mathbb{R}^+ \times [0, 1] \rightarrow \mathbb{R}^+$  gives the flow of interviews between a firm located at f and a worker located at w. As usual, m(v(f), u(w)) is increasing in both v(f) and u(w), and is constant returns to scale. Let q(f, w) = m(v(f), u(w))/v(f) be the probability that a firm at f meets a worker from w, and  $\theta(f, w) = v(f)/u(w)$  is the corresponding labor market tightness. Since mismatch

<sup>&</sup>lt;sup>12</sup>In the model environment, location is synonymous with skills. A worker at location w is equivalent to a worker with skill w. Similarly, a firm at location f is same as a job with skill requirement f.

is costly in our environment ( $\gamma > 0$ ), only a fraction of these meetings with sufficiently low mismatch will materialize into productive job matches, and this fraction will be determined as a part of the equilibrium.

## **3.4 Continuation Values**

In this section, I present the recursive formulation of the dynamic problem faced by the firms and the workers. Let V(f) denote the value of a vacant firm at location f.

$$rV(f) = -c + \frac{1}{2\pi} \int_{f}^{f+2\pi} q(f,\tau) \max\{J^{0}(f,\tau), V(f)\} d\tau,$$
(4)

where r is the interest rate. A vacant firm at location f upon paying a cost c to post a vacancy, meets a worker from location  $\tau$  with probability  $q(f, \tau)$ . Upon meeting, the vacant firm must decide whether to accept the match, earning value  $J^0$  or continue to remain vacant. Assuming there is a free entry of vacancies, this will drive down the value of all the vacancies to zero in equilibrium.

$$V(f) = 0, \forall f \in [0, 2\pi].$$
 (5)

The continuation value of an unemployed worker, U is given by

$$rU(w) = b + \frac{1}{2\pi} \int_{w}^{w+2\pi} \theta(\tau, w) q(\tau, w) \Big[ \max\{W^{0}(\tau, w), U(w)\} - U(w) \Big] d\tau,$$
(6)

where *b* is the flow value of unemployment. The unemployed worker at location *w* meets a firm from  $\tau$  with a probability  $\theta(\tau, w)q(\tau, w)$ , and has to decide whether to accept the job match and earn a value of  $W^0$  or continue to remain unemployed.

We now move onto the continuation values during the period of match creation. The value the firm receives at the period of match formation is given by

$$rJ^{0}(f,w) = \eta(f,w)\bar{\epsilon} - \omega_{0}(f,w) + \lambda \int_{0}^{\bar{\epsilon}} \left[ \max\{J(f,w,z), V(f)\} - J^{0}(f,w) \right] dF(z).$$
(7)

The total output of a match is given by the product of mismatch component of the productivity  $\eta(f, w)$  and the idiosyncratic productivity realization  $\epsilon$ . Following other

models of endogenous job separation like Mortensen and Pissarides (1994), Mortensen and Pissarides (1999), and Fujita and Ramey (2012), new matches are formed at the frontier of the idiosyncratic productivity distribution  $\bar{\epsilon}$ . After the starting period, matches are subject to productivity shocks with probability  $\lambda$  and the new productivity realization is drawn from the distribution F.  $\omega_0$  is the wage paid to the worker at the period of match creation, and the wages are determined by Nash bargaining as explained in the next section. Similarly, the continuation value of the worker at the starting period is given by

$$rW^{0}(f,w) = \omega_{0}(f,w) + \lambda \int_{0}^{\bar{\epsilon}} \left[ \max\{W(f,w,z), U(w)\} - W^{0}(f,w) \right] dF(z).$$
 (8)

The worker at the starting period receives a wage  $\omega_0$  and has to decide whether to continue with the match under the new productivity realization, earning a value of W or become unemployed earning a value of U. Finally, I list the continuation values of firms and workers that are a part of the existing matches. The only difference is, the matches need not be at the highest idiosyncratic productivity level, and hence the values depend on the productivity realization.

$$rJ(f,w,\epsilon) = \eta(f,w)\epsilon - \omega(f,w,\epsilon) + \lambda \int_0^{\bar{\epsilon}} \left[ \max\{J(f,w,z), V(f)\} - J(f,w,\epsilon) \right] dF(z).$$
(9)

As before, the productive firm earns output net of wages paid and has to decide whether to continue under the new productivity, earning a value of J or dissolve the match and become vacant. Similarly, the worker's continuation value is given by

$$rW(f, w, \epsilon) = \omega(f, w, \epsilon) + \lambda \int_0^{\overline{\epsilon}} \left[ \max\{W(f, w, z), U(w)\} - W(f, w, \epsilon) \right] dF(z).$$
(10)

## 3.5 Wage Determination

The surplus generated from a successful match is shared between the worker and the firm using Nash bargaining. The wage of a worker having bargaining power  $\beta$  satisfies

the equation

$$(1-\beta)\Big[W(f,w,\epsilon) - U(w)\Big] = \beta J(f,w,\epsilon).$$
(11)

Substituting the definition of continuation values, the wage received by a matched worker is given by

$$\omega(f, w, \epsilon) = \beta \left[ \eta(\widehat{f, w})\epsilon + \frac{1}{2\pi} \int_{w}^{w+2\pi} \theta(\tau, w)q(\tau, w)J^{0}(\tau, w)\,d\tau \right] + (1 - \beta)b, \tag{12}$$

and the starting wage is given by

$$\omega_0(f,w) = \beta \left[ \eta(\widehat{f,w})\overline{\epsilon} + \frac{1}{2\pi} \int_w^{w+2\pi} \theta(\tau,w)q(\tau,w)J^0(\tau,w)\,d\tau \right] + (1-\beta)b. \tag{13}$$

## 3.6 Equilibrium

Following Marimon and Zilibotti (1999), Gautier et al. (2006), and Gautier et al. (2010), I solve for the symmetric equilibrium where both unemployment and vacancies are uniformly distributed over the circle. The following proposition proves that it is an equilibrium in our environment.

**Proposition 1.** Given a free entry of vacancies, a uniform distribution of unemployment and vacancies, i.e.  $u(w) = u \forall w \in [0, 2\pi]$  and  $v(f) = v \forall f \in [0, 2\pi]$  is an equilibrium.

### **Proof.** In Appendix C.

The intuition is, given that unemployment has a uniform distribution, vacancies must be distributed uniformly. Otherwise, in a location with relatively more vacancies, the outside option of being unemployed (and hence wages) will be higher. This reduces the value of vacancies at such locations, which in turn violates the free entry condition. Similarly, given that vacancies are uniformly distributed, unemployment must also have a uniform distribution. If not, firms can profitably deviate and post vacancies at the location having more unemployment, again violating the free entry condition.

The major implication of this proposition is that market tightness  $\theta$  no longer de-

pends on the location of the match i.e.,  $\theta(f, w) = \theta, \forall f, w \in [0, 2\pi]$ . As a result, continuation values depend only on the distance between the firm and the worker in the match i.e.,  $x \equiv \widehat{f, w}$  and not on the location per se. Under a uniform distribution of unemployment and vacancies, we can simplify the continuation values as follows. The continuation value of a vacant firm is given by

$$rV = -c + 2q(\theta) \int_0^{\bar{x}} J^0(\tau) \, d\tau,$$
(14)

while that of an unemployed worker is

$$rU = b + 2\theta q(\theta) \int_0^{\bar{x}} \left[ W^0(\tau) - U \right] d\tau.$$
(15)

Here,  $\bar{x}$  denotes the cut-off distance between a worker and a firm. If the distance is greater than  $\bar{x}$ , workers and firms choose to walk away during the interview without forming a match. Continuation values during the time of the match can be reformulated as

$$rJ^0(x) = \eta(x)\bar{\epsilon} - \omega_0(x) + \lambda \int_0^{\bar{\epsilon}} \left[ \max\{J(x,z),V\} - J^0(x) \right] dF(z),$$
(16)

while that of the worker is

$$rW^{0}(x) = \omega_{0}(x) + \lambda \int_{0}^{\bar{\epsilon}} \left[ \max\{W(x,z),U\} - W^{0}(x) \right] dF(z).$$
(17)

Finally, the continuation value of a firm involved in an existing match with idiosyncratic productivity  $\epsilon$  is given by

$$rJ(x,\epsilon) = \eta(x)\epsilon - \omega(x,\epsilon) + \lambda \int_0^{\overline{\epsilon}} \left[ \max\{J(x,z),V\} - J(x,\epsilon) \right] dF(z),$$
(18)

while that of the worker is

$$rW(x,\epsilon) = \omega(x,\epsilon) + \lambda \int_0^{\bar{\epsilon}} \left[ \max\{W(x,z),U\} - W(x,\epsilon) \right] dF(z).$$
(19)

The equilibrium wage in the starting period simplifies to

$$\omega_0(x) = \beta[\eta(x)\bar{\epsilon} + c\theta] + (1-\beta)b, \tag{20}$$

and the continuing wages are given by

$$\omega(x,\epsilon) = \beta[\eta(x)\epsilon + c\theta] + (1-\beta)b.$$
(21)

## 3.7 Model Solution

Equilibrium of this model is characterized by the labor market tightness  $\theta$ , cutoff mismatch  $\bar{x}$ , and the cutoff productivity schedule  $\epsilon^*(x)$ . The cutoff mismatch  $\bar{x}$  determines the proportion of interviews that gets converted into productive matches. If the distance between the interviewing worker and the firm is greater than  $\bar{x}$ , both the worker and the firm choose to abandon the interview without forming a match. Analogously, the cutoff productivity  $\epsilon^*(x)$  determines the fraction of existing matches with mismatch x that will continue into the future. If the idiosyncratic productivity of a match with mismatch x is below  $\epsilon^*(x)$ , both the firm and the worker mutually choose to separate from the existing match. I make use of the free entry condition and the definition of cutoffs to solve for the equilibrium objects.

### **Free Entry Condition**

With a free entry of vacancies, the value of a vacant firm is zero in equilibrium.

$$rV = 0. (22)$$

Using the definition of continuation values and wages, we get the following equation

$$c = \frac{2q(\theta)(1-\beta)}{r+\lambda} \left[ \int_0^{\bar{x}} \eta(x)\bar{\epsilon}\,dx - b\bar{x} - \frac{\beta c\theta\bar{x}}{1-\beta} + \frac{\lambda}{r+\lambda} \int_0^{\bar{x}} \int_{\epsilon^*(\tau)}^{\bar{\epsilon}} \eta(\tau)(z-\epsilon^*(\tau))\,dF(z)\,d\tau \right].$$
(23)

### **Cutoff Mismatch**

The cutoff distance  $\bar{x}$  gives the level of mismatch at which the meeting firm and the worker are indifferent between forming the match and walking away empty handed.

$$W^{0}(\bar{x}) - U = J^{0}(\bar{x}) = 0.$$
(24)

Substituting the continuation values, we get

$$\bar{\epsilon} + \frac{\lambda}{r+\lambda} \int_{\epsilon^*(\bar{x})}^{\bar{\epsilon}} (z - \epsilon^*(\bar{x})) \, dF(z) = \frac{b}{\eta(\bar{x})} + \frac{\beta c\theta}{(1-\beta)\eta(\bar{x})}.$$
(25)

## **Cutoff Productivity**

Idiosyncratic productivity at the cutoff level  $\epsilon^*(x)$  leaves an existing match with mismatch x indifferent between continuing to stay together and ending the match.

$$W(x, \epsilon^*(x)) - U = J(x, \epsilon^*(x)) = 0.$$
(26)

Substituting the continuation values, we have

$$\epsilon^*(x) + \frac{\lambda}{r+\lambda} \int_{\epsilon^*(x)}^{\bar{\epsilon}} (z-\epsilon^*(x)) \, dF(z) = \frac{b}{\eta(x)} + \frac{\beta c\theta}{(1-\beta)\eta(x)}.$$
(27)

Equations (23), (25), and (27) constitute the equilibrium conditions of the model and they are solved simultaneously to obtain the equilibrium  $\theta$ ,  $\bar{x}$ , and  $\epsilon^*(x)$ . The derivations of these conditions are given in appendix **C**.

## 3.8 Labor Market Flows

In this section, I present the equations governing the labor market flows. Even though we do not need to obtain the employment distribution to solve the model, these distributions are needed to calculate labor market turnover, the primary objective of this paper. Let  $e_x(\epsilon)$  represent the distribution (CDF) of employment with mismatch x. So, the total employment in the economy having mismatch of x is given by  $e_x(\bar{\epsilon})$ . Aggregate employment e is obtained by integrating over all possible mismatch values,

$$e = 2 \int_0^{\bar{x}} e_x(\bar{\epsilon}) \, dx. \tag{28}$$

Since there is a unit measure of workers in total, aggregate unemployment u is

$$u = 1 - e. \tag{29}$$

I now detail the flow equations of employment at each level of mismatch x.

Inflow into unemployment from 
$$x = \lambda F(\epsilon^*(x)) e_x(\bar{\epsilon})$$
. (30)

The measure of workers who transition from being employed with mismatch x to unemployment is the fraction of total employment with mismatch x who receives a productivity realization lower than the cutoff productivity  $\epsilon^*(x)$ .

Outflow from unemployment to 
$$x = \theta q(\theta)u$$
. (31)

The probability of an unemployed worker finding a job with mismatch x is fairly standard. Since we consider an equilibrium with a uniform distribution of vacancies and unemployment, market tightness  $\theta$  and hence the job finding probability  $\theta q(\theta)$  does not depend on the location of job creation.

In the steady state, the inflows into unemployment should be equal to the outflows from unemployment at each mismatch level x. Equating the flow equations (30) and (31) gives us an expression for the total employment at each x.

$$e_x(\bar{\epsilon}) = \frac{\theta q(\theta) \left[ 1 - 2 \int_0^{\bar{x}} e_x(\bar{\epsilon}) dx \right]}{\lambda F(\epsilon^*(x))}.$$
(32)

Once we have the total employment for each mismatch level, we can retrieve the distribution of employment over the space of x and  $\epsilon$  as follows.

$$e_{x}(\epsilon) = \begin{cases} 0 & \text{if } \epsilon < \epsilon^{*}(x) \\ \left[F(\epsilon) - F(\epsilon^{*}(x))\right] e_{x}(\bar{\epsilon}) & \text{if } \epsilon^{*}(x) \leq \epsilon < \bar{\epsilon} \\ \frac{\theta q(\theta) \left[1 - 2\int_{0}^{\bar{x}} e_{x}(\bar{\epsilon}) \, dx\right]}{\lambda F(\epsilon^{*}(x))} & \text{if } \epsilon = \bar{\epsilon} \end{cases}$$

Employment at each mismatch level x exists in the interval  $[\epsilon^*(x), \bar{\epsilon}]$ . The above equation for distribution is obtained by equating the flows in and out of employment at each value of x and  $\epsilon$ . Finally, we can derive the aggregate separation rate (s), one of the measures of labor market turnover. It is defined as the total job separations across all mismatch levels as a fraction of aggregate employment.

$$s = \frac{\lambda \int_0^x F(\epsilon^*(x)) e_x(\bar{\epsilon}) \, dx}{\int_0^{\bar{x}} e_x(\bar{\epsilon}) \, dx}.$$
(33)

The other measure of labor market turnover, job finding rate ( $\varphi$ ) is defined as the total outflows from unemployment to employment at every mismatch level over the circle as a fraction of aggregate unemployment.

$$\varphi = 2\bar{x}\theta q(\theta). \tag{34}$$

# 4. Calibration

I calibrate the model to quantitatively assess the impact of an increase in job specialization on labor market turnover. In total, there are 10 parameters to calibrate. Three parameters are chosen externally outside the model while the remaining seven parameters are selected so that the model can match various moments from the data.

The top panel of table 5 gives the values of the parameters that are chosen externally without solving the model. The model is calibrated at a monthly frequency. The interest rate r is set to 0.004 to obtain an annual interest rate of 4.8%. The matching function is assumed to be Cobb-Douglas of the form  $m = \mu u^{\alpha} v^{1-\alpha}$ . The elasticity of matching function with respect to unemployment,  $\alpha$  is chosen to be 0.5 following the evidence reported in Petrongolo and Pissarides (2001). Following most of the literature such as Pissarides (2009) and Fujita (2018), worker's bargaining power  $\beta$  is set equal to the elasticity of matching function.

The strategy followed to calibrate the rest of the model parameters is shown in the bottom panel of table 5. The parameters are chosen by minimizing the distance between the model's initial steady state moments and their corresponding data counterparts. Following Fujita and Ramey (2007) and Fujita (2018), vacancy posting cost c is chosen to achieve the firm meeting rate,  $q(\theta)$  of 0.9. The flow value of unemployment b is set to 70% of the aggregate wage. This replacement ratio is closer to the values used by Hall and Milgrom (2008) and Fujita and Ramey (2012).<sup>13</sup> The effi-

<sup>&</sup>lt;sup>13</sup>Shimer (2005) calibrates the value of *b* using a replacement ratio of 40%, while Hagedorn and Manovskii (2008)'s calibration implies a replacement ratio of 95.5%. Majority of the literature reconciles both the strategies and uses a replacement ratio of around 70%.

Parameter Definition		Value	Source/Target
Chosen Externally			
,			
Interest rate	r	0.004	Period = Month
Worker's bargaining power	$\beta$	0.5	
Elasticity of matching function	$\alpha$	0.5	
Chosen Internally			
Efficiency of matching function	$\mu$	0.767	Job finding rate ( $\varphi$ )
Frequency of shocks	$\lambda$	0.10	Separation rate (s)
Vacancy posting cost	c	1.01	Firm meeting rate (q)
Flow value of unemployment	b	0.656	Replacement ratio
Std. dev. of shocks	$\sigma_{\epsilon}$	0.3918	Employment share at $x = 0.1$
Maximum shock realization	$\overline{\epsilon}$	1.7014	Std. dev. of employment distribution
Importance of mismatch	$\gamma$	0.573	Job specialization ( $\phi$ )

Table 5: Calibration

ciency parameter of the matching function  $\mu$  is set to target the aggregate job finding rate corresponding to the initial steady state. The target job finding rate is chosen to be 0.44 consistent with evidence from the CPS microdata over the period 1978–1995.

The idiosyncratic productivity process follows a truncated lognormal distribution and has three parameters to be calibrated. The frequency of arrival of idiosyncratic productivity shocks  $\lambda$  is chosen to match the aggregate separation rate in the initial steady state. The target separation rate is set to 4.2%, consistent with the CPS evidence during 1978–1995. The standard deviation of the idiosyncratic productivity realization  $\sigma_{\epsilon}$  is selected to match the employment share at the low mismatch level of 0.1 during 1978–1995. The upper support of productivity realizations  $\bar{\epsilon}$  is chosen to match the standard deviation of employment distribution over mismatch during the same period.

Finally, the parameter governing the impact of mismatch on productivity,  $\gamma$  is calibrated by matching the estimate of job specialization  $\phi$  obtained from regression (2)

Moments		Target	Model	Source
Job finding rate	$\varphi$	0.4412	0.4429	CPS (1978–1995)
Separation rate	s	0.0416	0.0426	CPS (1978–1995)
Firm meeting rate	q	0.90	0.9142	Fujita and Ramey (2007)
Replacement ratio		0.70	0.67	Fujita (2018)
Emp. share at $x = 0.1$		0.3692	0.3666	NLSY79 (1978–1995)
Std. dev. of emp. distribution		0.1642	0.1480	NLSY79 (1978–1995)
Job specialization	$\phi$	-0.3190	-0.3185	NLSY79 (1978–1995)

Table 6: Matching the Targets

with the model counterpart given by coefficient  $\delta$  in the following regression

$$\ln \omega(x,\epsilon) = \beta_0 + \delta x + \beta_1 \epsilon + e, \tag{35}$$

with  $x \leq \bar{x}$  and  $\epsilon \geq \epsilon^*(x)$ .  $\omega(x, \epsilon)$  is the equilibrium wage earned by the worker in a job with mismatch x and facing an idiosyncratic productivity realization of  $\epsilon$ .  $\delta$  captures the wage loss associated with mismatch and is the model analog of job specialization  $\phi$  estimated from the data. Table 6 shows the performance of the model in matching the chosen targets. Overall, the model is successful in generating the steady state moments that are close to their data counterparts. In the quantitative analysis below, I vary the mismatch parameter  $\gamma$  to study the impact of increase in job specialization on labor market turnover.

# 5. Results

We now turn to the main question of the paper. How does an increase in job specialization affect labor market turnover? To answer this question, I vary the mismatch parameter  $\gamma$  to capture the increase in job specialization estimated from NLSY data. Table 7 shows the new steady state associated with higher job specialization. Data moments refer to the IV-OLS estimate of job specialization from the wage regression (2), while the model moment is the corresponding estimate from regression (35). By

$\gamma$	Data	Model	Source
0.5730	-0.3190	-0.3185	NLSY79 (1978–1995)
1.0003	-0.5768	-0.5767	NLSY97 (1996–2014)

Table 7: Increase in Specialization

*Note:* This table shows the change in mismatch parameter  $\gamma$  to capture the increase in job specialization observed in the data. The data moment is the IV-OLS estimate of job specialization obtained from wage regression (2) while the model moment is the corresponding estimate from regression (35).

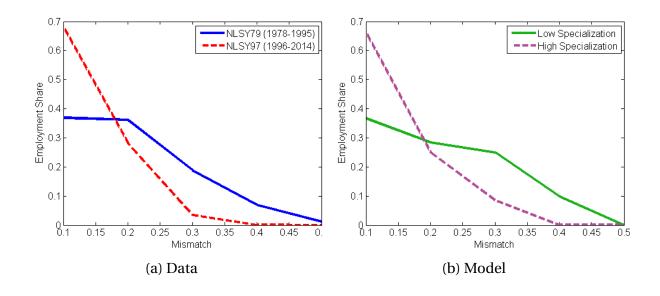
analyzing the model behavior under these two steady states, we can understand the impact of higher job specialization on labor market turnover.

## 5.1 Employment Distribution

We start our discussion by examining the impact of increased specialization on the employment distribution. Figure 4a shows the empirical distribution obtained from NLSY79 and NLSY97. As discussed earlier, over time, more matches have moved to-ward lower mismatch, thus leading to increased sorting in the labor market. Figure 4b shows the steady state distribution generated by the model under low and high specialization, where the increase in specialization is calibrated to match the data.

Consistent with the data, the model is successful in generating a monotonically decreasing distribution, with the number of matches declining with mismatch. More importantly, with the increase in specialization, the model is successful in replicating the empirical change in the distribution. With higher specialization, almost 65% of the matches are formed with mismatch less than or equal to 0.1, compared to about 35% in the case of low specialization. Since mismatch has become more costly, high mismatch jobs are no longer sustainable, and hence majority of employment shifts toward lower mismatch.

Even though a number of recent studies like Barth et al. (2016) and Song et al. (2019) document an increase in labor market sorting for the US, Card et al. (2013) for Germany, and Håkanson et al. (2020) for Sweden, the underlying reasons for this



*Note:* Panel (a): Empirical distribution of employment over mismatch obtained from NLSY79 and NLSY97. Panel (b): Corresponding employment distribution generated by the model under low and high specialization respectively.

### Figure 4: Employment Distribution

change is still being explored. The current paper contributes to this literature by showing that increase in the specialization of jobs is an important contributor for the observed rise in sorting found in the data. With increased cost of mismatch, firms and workers reduce their mismatch during match formation, thus leading to improved sorting in the labor market.

## 5.2 Labor Market Turnover

The main results of the paper showing the relationship between job specialization and labor market turnover are summarized in table 8. Low specialization represents the initial steady state, and it is calibrated to match the job finding and separation rates in the earlier period of 1978–1995. The  $\delta$  parameter reflects the wage loss associated with mismatch estimated using regression (35).The model does a good job of matching the job specialization and turnover measures from the data. The steadystate unemployment rate generated by the model is around 8.9%, while the equilibrium cutoff mismatch is 0.34. This shows that the skills are not perfectly substi-

	δ	$\varphi$	s	θ	u	$\bar{x}$
Low Specialization	-0.3185	0.4429	0.0426	0.7040	0.0886	0.3441
High Specialization	-0.5767	0.2951	0.0270	0.6273	0.0849	0.2429

Table 8: Effects of Increase in Specialization

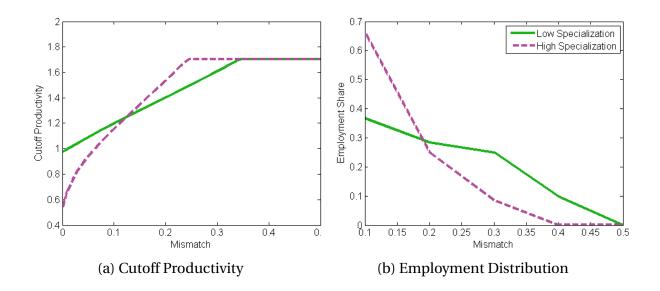
*Note:* Increase in specialization captured by  $\delta$  estimated from regression (35).  $\varphi$ : job finding rate, *s*: separation rate,  $\theta$ : labor market tightness, *u*: unemployment rate,  $\bar{x}$ : cutoff value of mismatch.

tutable, as even under the highest productivity realization, a worker is suitable to work at only about 70% of the jobs.

With increase in specialization as shown by the increase in cost of mismatch  $\delta$ , the model is successful in generating a decline in both the measures of labor market turnover. As specialization increases, the job finding rate  $\varphi$  decreases from 0.44 to around 0.30, while the separation rate reduces from 0.043 to 0.027. Unemployment rate goes down slightly from 8.9% to 8.5%, and the labor market tightness reduces from 0.70 to 0.63. Finally, the cutoff mismatch also decreases from 0.34 to 0.24. Thus, with the increase in specialization, the labor market effectively faced by a worker (or a firm) has narrowed down, as each worker is suitable to work in only about 50% of the jobs, compared to about 70% in the initial steady state. This narrowing of the labor market is an important channel causing the observed decline in the labor market turnover as described next.

### **Separation Rate**

As the job specialization increases, the cost of mismatch goes up, and hence the substitutability between the skills reduces. This causes the workers and firms who are already well-matched to become more reluctant to separate from each other. With the narrowing of the labor market, these well-matched firms and workers realize that, once separated from their existing matches, it is more difficult to get a better match in the future. This causes the cutoff productivity, and hence the separation rate of these good matches to fall. Figure 5a shows the cutoff productivity as a function of



*Note:* Panel (a): Cutoff productivity as a function of mismatch under low and high specialization. The level of mismatch where the schedule flattens represents the cutoff mismatch. Panel (b): Model generated employment distribution as a function of mismatch under low and high specialization.

#### Figure 5: Mechanism

mismatch. Well-matched workers and firms prefer to continue in their matches for a longer period of time, and hence endure much lower idiosyncratic productivity realizations compared to matches with higher mismatch. This causes the cutoff productivity to be an increasing function of mismatch as long as the mismatch is less than the cutoff level, beyond which the productivity schedule flattens. As the specialization increases, the cutoff productivity schedule pivots to the left as seen in figure 5a. This causes the cutoff productivity of matches with low mismatch to decrease, leading to a further decline in the separation rate of these good matches. On the other hand, with an increase in specialization, the cost of mismatch has increased, making matches with high mismatch even more difficult to sustain at lower levels of productivity. This causes the cutoff productivity of these matches to increase. Hence, firms and workers involved in the bad matches are more likely to separate now than before, leading to an increase in the separation rate.

To disentangle which effect has the bigger impact, I look at the distribution of employment over mismatch. Figure 5b shows the employment distribution under low

and high specialization. As discussed in the previous section, increase in specialization increases the cost of mismatch, thus making firms and workers sort themselves into better matches. This leads to a shift in the composition of employment towards lower mismatch. With this shift in composition, majority of the matches faces lower separation rate, while a minority faces higher separation rate, leading to a decline in the aggregate separation rate.

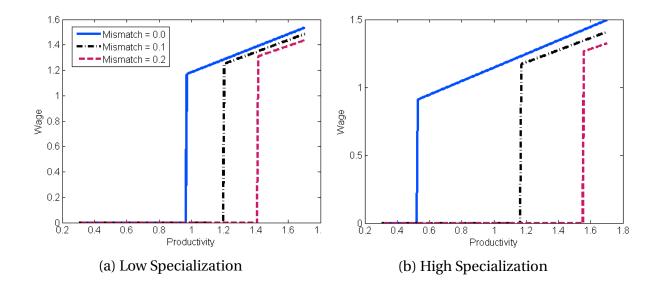
### **Job Finding Rate**

Unlike the separation rate, every unemployed worker faces the same job finding rate  $\varphi$ , and this rate depends positively on labor market tightness  $\theta$  and cutoff mismatch  $\bar{x}$ . As seen from table 8, increase in specialization reduces the tightness as well as the cutoff mismatch, and hence the job finding rate. Increase in specialization narrows the labor market, making it more difficult for a firm to find a suitable worker. This reduces incentive for the firms to post vacancies, as total vacancies reduce by around 15%, leading to a reduction in the labor market tightness.<sup>14</sup> With  $\bar{x}$  declining, both firms and workers are forced to be matched in a narrower region of the labor market compared to the earlier times. Thus, with higher specialization, firms and workers become more selective in accepting a match, leading to a decline in the job finding rate.

## 5.3 Wage Dispersion

Apart from the steady decline in labor market turnover, another labor market trend that has garnered a lot of research interest is the increase in wage dispersion (Katz and Murphy (1992); Murphy and Welch (1992); Juhn et al. (1993); Autor et al. (2008)). Figure 6a shows the steady state wage generated by the model under low specialization for three different levels of mismatch, while figure 6b gives the corresponding wage function when the specialization is high. As expected, wages are increasing with productivity, but decreasing with mismatch. The vertical lines correspond to the cutoff productivity for a given level of mismatch. More importantly, one can see

<sup>&</sup>lt;sup>14</sup>The total vacancies  $v = \theta \times u$ . Under low specialization,  $v = 0.7040 \times 0.0886 = 0.0624$ . With increase in specialization,  $v = 0.6273 \times 0.0849 = 0.0533$ , translating to a decline of 14.6%.



*Note:* Wages as a function of productivity for different levels of mismatch. The vertical lines correspond to the cutoff productivity for a given level of mismatch.

Figure 6: Wage Function

that, with increased specialization, the dispersion of wages for a given productivity level has increased. Under low specialization, the workers who were matched perfectly (mismatch of 0) were earning around 6.5% more on average, compared to those with a mismatch of 0.2. With increased specialization, this wage premium enjoyed by workers with zero mismatch has jumped to around 13% on average. To characterize the wage dispersion, Hornstein et al. (2011) propose the mean-min ratio of the wage distribution as a useful measure. Calculating this statistic for our steady state distributions, with higher specialization, the mean-min ratio goes up from 1.17 to 1.39. Thus, increase in job specialization leads to higher wage dispersion, measured either in terms of relative wages, or the mean-min ratio.

Studies like Song et al. (2019), Barth et al. (2016), and Card et al. (2013) decompose the increase in wage dispersion and show that, increased sorting in the labor market is an important contributor for this increase. One explanation proposed for this increased wage dispersion is skill-biased technical change. Acemoglu and Autor (2011) provide a detailed overview of this literature. Another reason that has been analyzed in the literature is increased outsourcing. Handwerker (2017) show that increased

outsourcing in the US has contributed to an increase in wage inequality, while Goldschmidt and Schmieder (2017) make a similar argument for Germany. I contribute to this literature by showing that increase in job specialization could be an important channel for the increase in wage inequality. Consistent with the empirical findings of Barth et al. (2016) and Song et al. (2019), the current paper shows that increased job specialization can generate increased sorting and higher wage dispersion in the labor market.

### 5.4 Average Labor Productivity

Finally, I investigate the impact of increase in job specialization on the average productivity of active matches. The increase in specialization makes mismatch more costly, thus negatively affecting the productivity. On the other hand, increased specialization also causes firms and workers to sort better, leading to an increase in productivity. Using the calibrated model, I find that, the first effect dominates the second effect, with labor productivity declining by 33.5% across the steady states. If we interpret this decline as happening over the period 1978–2014, it translates to a 1.1% decrease in the yearly growth rate of productivity. This is consistent with the findings of Byrne et al. (2016), who document a slowdown in labor productivity growth in the US after 2000. Even though increased specialization causes workers and firms to move to better matches (resulting in lower mismatch), the cost of mismatch has increased even more, leading to a decline in the labor productivity growth.

# 6. Conclusion

This paper argues that specialization of jobs has increased over time, and this can explain the decline in both job finding and separation rates. Job specialization is measured as the cost of mismatch on match productivity, where mismatch is the distance between the skills of a worker and the skill requirements of their job. I estimate job specialization using individual level data from NLSY79 and NLSY97, and show that the estimated job specialization has increased after 1995. To understand the implications of this increase in specialization, I construct an equilibrium labor search model

with ex-ante heterogeneous firms and workers. Calibrating this model to the US, I find that the increase in specialization is an important source of decline in the labor market turnover. As jobs get more specialized, both firms and workers become more selective in their match formation. This causes the good matches to last longer, while bad matches get destroyed faster compared to before. Since increased specialization also leads to a shift in the composition of employment toward better matches, there is a reduction in the aggregate turnover of the economy. Along with the decline in labor market turnover, the increase in specialization can also explain the increase in wage dispersion, and the slowdown in labor productivity growth. Further investigating the underlying determinants of this increase in job specialization, and analyzing their implications for labor market dynamics could be a fruitful area of research.

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# Appendix

# A. Data

### A.1 Sample Selection

To estimate the change in job specialization, I obtain individual level data from NLSY79 and NLSY97. Specifically, I use observations from NLSY79 for 1978–1995, while the data for 1996–2014 is obtained from NLSY97. NLS data provides employment history for each individual over their labor market experience. The primary job for each individual is identified as one where the individual has spent the maximum amount of time in the given year.

I consider the cross-sectional sample of both NLSY79 and NLSY97. I drop all the individuals who have worked for more than 1200 hours during the initial year of the survey. This limits the analysis to those individuals who enter the labor market after the start of the survey. I also consider only those individuals who have worked more than 1200 hours for at least two consecutive years. The individuals who do not have valid ASVAB scores or demographic information are also dropped from the sample. Wages from both NLSY79 and NLSY97 are converted to 2009 dollars using PCE deflator. Following Deming (2017), I drop observations with real wages below 3 or above 200 dollars to reduce the effect of outliers. Finally, I restrict the sample to individuals aged between 16 and 35 years to maximize the comparability between NLSY79 and NLSY97 cohorts. At the end of this process, the NLSY79 sample has 44,886 individual-year observations over 1978–1995, while the corresponding NLSY97 sample has 41,864 individual-year observations over 1996–2014. The descriptive statistics of both NLSY79 and NLSY97 samples are given in table A1.

### A.2 Occupation and Industry Codes

The occupations in NLSY79 from 1978 to 1995 are represented using 1970 Census Occupation Classification. The corresponding occupations in NLSY97 from 1996 to 2014 are coded using 2002 codes. To make the occupational classifications com-

NLSY79	NLSY97
1978–1995	1996–2014
44,886	41,864
24.86	23.75
51.76%	48.42%
59.61%	54.27%
22.41%	25.29%
17.98%	20.44%
7.34%	13.88%
12.38%	15.11%
7.40	5.14
2.71	2.39
3.37	2.68
13.97	15.46
1555.41	1470.21
	1978–1995 44,886 24.86 51.76% 59.61% 22.41% 17.98% 7.34% 12.38% 7.40 2.71 3.37 13.97

# Table A1: Descriptive Statistics

parable, I convert both 1970 and 2002 codes into 1990 occupational codes using crosswalks developed by Autor and Dorn (2013) and accessed from https://www. ddorn.net/data.htm. Similarly, industry codes vary between NLSY79 and NLSY97. The industries in NLSY79 are coded using 1970 Census Industry Classification, while NLSY97 uses 2002 Census codes. I make use of the crosswalk developed by Guvenen et al. (2020) (table C.4) to convert the industry codes in both NLSY79 and NLSY97 into 1-digit 1970 Census codes. Finally, I streamline the industry and occupation codes within each employment spell to remove any spurious changes. For each employment spell, I replace the industry and occupation codes with the one that is reported most number of times within the spell.

# **B.** Supplementary Empirical Evidence

### **B.1 Regression Tables**

	NLSY79 (1	978–1995)	NLSY97 (1996–2014	
	(1)	(2)	(3)	(4)
	OLS	IV-OLS	OLS	IV-OLS
Mismatch	-0.3051***	-0.3190***	-0.5836***	-0.5768***
	(0.0795)	(0.0818)	(0.0824)	(0.0866)
Worker Skill (Average)	0.2963***	0.3035***	0.1225***	0.0713*
	(0.0448)	(0.0493)	(0.0367)	(0.0411)
Occ. Requirement (Average)	0.1846***	0.0642	0.2839***	0.2235***
	(0.0477)	(0.0516)	(0.0442)	(0.0509)
Skill $\times$ Occ. Tenure	0.0365***	0.0345***	0.0138	0.0342***
	(0.0095)	(0.0088)	(0.0120)	(0.0091)
Requirement × Occ. Tenure	0.0214***	0.0508***	0.0472***	0.0705***

Table B1: Job Specialization in NLSY79 and NLSY97

	(0.0077)	(0.0072)	(0.0114)	(0.0085)
Experience	-0.0079	0.0285***	0.0249***	0.0866***
	(0.0049)	(0.0053)	(0.0042)	(0.0057)
Experience <sup>2</sup>	0.0032***	0.0007	0.0005	-0.0040***
	(0.0007)	(0.0006)	(0.0006)	(0.0007)
Experience <sup>3</sup>	-0.0001***	-0.0000	-0.0001**	0.0001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Job Tenure	0.0501***	-0.0047	0.0281*	-0.0111
	(0.0097)	(0.0099)	(0.0147)	(0.0120)
Job Tenure <sup>2</sup>	-0.0081***	-0.0003	-0.0065**	0.0002
	(0.0017)	(0.0015)	(0.0030)	(0.0021)
Job Tenure <sup>3</sup>	0.0004***	0.0000	0.0003**	0.0000
	(0.0001)	(0.0001)	(0.0002)	(0.0001)
Occ. Tenure	0.0230**	-0.0139	0.0162	-0.0606***
	(0.0100)	(0.0102)	(0.0140)	(0.0132)
Occ. Tenure <sup>2</sup>	-0.0017	-0.0023*	0.0002	0.0003
	(0.0015)	(0.0012)	(0.0026)	(0.0020)
Occ. Tenure <sup>3</sup>	-0.0000	0.0001	-0.0001	-0.0000
	(0.0001)	(0.0000)	(0.0001)	(0.0001)
Old Job	-0.0274***	-0.0249***	-0.0084	-0.0152***
	(0.0064)	(0.0061)	(0.0080)	(0.0057)
Female	-0.1174***	-0.1152***	-0.1266***	-0.1298***
	(0.0093)	(0.0097)	(0.0095)	(0.0102)
Hispanic	0.0270	0.0321*	0.0349***	0.0481***

	(0.0181)	(0.0191)	(0.0129)	(0.0142)
Black	-0.0528***	-0.0529***	-0.0118	-0.0053
	(0.0128)	(0.0136)	(0.0120)	(0.0128)
< College	0.1296***	0.1189***	0.0598***	0.0314**
	(0.0128)	(0.0135)	(0.0116)	(0.0128)
College	0.3295***	0.3156***	0.2450***	0.1994***
	(0.0168)	(0.0177)	(0.0165)	(0.0180)
> College	0.3745***	0.3519***	0.2935***	0.2431***
	(0.0245)	(0.0251)	(0.0191)	(0.0207)
Constant	1.8700***	1.9127***	2.0736***	2.1278***
	(0.0508)	(0.0540)	(0.0685)	(0.0706)
N	44886	44886	41864	41864

All regressions include industry and occupation fixed effects.

Standard errors in parentheses are clustered at individual level.

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

	NLSY79 (1978–1995)		NLSY97 (1996–2014	
	NL5179(1	970-1993)	NL5197 (1	990-2014)
	(1)	(2)	(3)	(4)
	GLS	IV-GLS	GLS	IV-GLS
Mismatch	-0.2294***	-0.2404***	-0.6561***	-0.6643***
	(0.0482)	(0.0498)	(0.0502)	(0.0523)
Worker Skill (Average)	0.3300***	0.3174***	0.2068***	0.1025**
	(0.0339)	(0.0427)	(0.0342)	(0.0449)
Occ. Requirement (Average)	0.1602***	0.0096	0.2718***	0.1863***
	(0.0316)	(0.0363)	(0.0307)	(0.0368)

Table B2: Job Specialization in NLSY79 and NLSY97

Skill × Occ. Tenure	0.0256***	0.0267***	0.0003	0.0284**
	(0.0053)	(0.0081)	(0.0072)	(0.0111)
Requirement $\times$ Occ. Tenure	0.0206***	0.0478***	0.0324***	0.0502***
-	(0.0045)	(0.0069)	(0.0066)	(0.0104)
Experience	0.0213***	0.0754***	0.0285***	0.1376***
	(0.0064)	(0.0080)	(0.0074)	(0.0101)
Experience <sup>2</sup>	0.0005	-0.0034***	0.0002	-0.0093***
Experience-	(0.0007)	(0.0008)	(0.0002)	(0.0011)
	(0.0001)	(0.0000)	(0.0000)	(0.0011)
Experience <sup>3</sup>	-0.0000*	0.0001***	-0.0000	0.0003***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Job Tenure	0.0153*	-0.0295**	0.0295***	-0.0342**
	(0.0084)	(0.0122)	(0.0100)	(0.0154)
Job Tenure <sup>2</sup>	-0.0028**	0.0027	-0.0066***	0.0025
,	(0.0013)	(0.0018)	(0.0019)	(0.0026)
Job Tenure <sup>3</sup>	0.0001**	-0.0001	0.0003***	-0.0001
job ichaic	(0.0001)	(0.0001)	(0.0001)	(0.0001)
000 700000	0.0112	0.0104*	0.0027	0.0400***
Occ. Tenure	0.0112	-0.0184*	-0.0027	-0.0469***
	(0.0074)	(0.0110)	(0.0099)	(0.0150)
Occ. Tenure <sup>2</sup>	0.0002	-0.0014	0.0043**	0.0012
	(0.0010)	(0.0014)	(0.0018)	(0.0023)
Occ. Tenure <sup>3</sup>	-0.0001*	0.0000	-0.0003***	-0.0001
	(0.0000)	(0.0001)	(0.0001)	(0.0001)
Old Job	-0.0180**	-0.0365***	-0.0363***	-0.0239*
·	(0.0078)	(0.0112)	(0.0083)	(0.0125)
Female	-0.0848***	-0.0791***	-0.0884***	-0.0880***

	(0.0039)	(0.0039)	(0.0039)	(0.0040)
Hispanic	0.0116*	0.0166**	0.0231***	0.0312***
	(0.0070)	(0.0071)	(0.0057)	(0.0058)
Black	-0.0415***	-0.0376***	-0.0132**	-0.0091
	(0.0060)	(0.0060)	(0.0054)	(0.0055)
< College	0.0940***	0.0839***	0.0260***	0.0148**
	(0.0068)	(0.0070)	(0.0063)	(0.0065)
College	0.2280***	0.2109***	0.1465***	0.1204***
	(0.0081)	(0.0083)	(0.0076)	(0.0079)
> College	0.2548***	0.2319***	0.1761***	0.1495***
	(0.0099)	(0.0102)	(0.0081)	(0.0084)
Constant	1.2396***	1.2172***	1.3854***	1.3066***
	(0.0280)	(0.0307)	(0.0337)	(0.0368)
Ν	35436	35436	32354	32354

All regressions include industry and occupation fixed effects.

Standard errors in parentheses are estimated using GLS with AR(1) error structure.

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Table B3: Job Specialization with less than High School Education
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	NLSY79 (1	NLSY79 (1978–1995)		996–2014)
	(1)	(1) (2)		(4)
	OLS	IV-OLS	OLS	IV-OLS
Mismatch	-0.1522	-0.1301	-0.4592***	-0.4371***
	(0.0961)	(0.1013)	(0.1020)	(0.1117)
Worker Skill (Average)	0.2385***	0.2275***	0.0043	-0.0180
	(0.0519)	(0.0607)	(0.0403)	(0.0511)

Occ. Requirement (Average)	-0.0215	-0.0631	0.1485***	0.1053
	(0.0565)	(0.0629)	(0.0542)	(0.0681)
Skill $\times$ Occ. Tenure	0.0383***	0.0403***	0.0106	0.0181
	(0.0132)	(0.0133)	(0.0177)	(0.0162)
Requirement $\times$ Occ. Tenure	0.0402***	0.0461***	0.0323	0.0608***
	(0.0111)	(0.0107)	(0.0201)	(0.0153)
Experience	-0.0021	0.0282***	0.0342***	0.0759***
Experience	(0.0065)	(0.0061)	(0.0049)	(0.0063)
<b>P</b> • 2				
Experience <sup>2</sup>	0.0027***	0.0008	-0.0007	-0.0033***
	(0.0010)	(0.0008)	(0.0008)	(0.0009)
Experience <sup>3</sup>	-0.0001***	-0.0000	-0.0000	0.0001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Job Tenure	0.0560***	0.0149	0.0751***	0.0406*
	(0.0135)	(0.0154)	(0.0203)	(0.0216)
Job Tenure <sup>2</sup>	-0.0054**	-0.0007	-0.0112***	-0.0050
	(0.0025)	(0.0025)	(0.0041)	(0.0039)
Job Tenure <sup>3</sup>	0.0001	-0.0000	0.0005**	0.0002
	(0.0001)	(0.0001)	(0.0002)	(0.0002)
Occ. Tenure	0.0088	-0.0436**	0.0035	-0.0689***
	(0.0139)	(0.0171)	(0.0184)	(0.0195)
Occ. Tenure <sup>2</sup>	-0.0037*	-0.0002	0.0000	-0.0004
oce. Ienure	(0.0022)	(0.0024)	(0.0032)	(0.0030)
	(0.0022)	(0.0024)	(0.0032)	(0.0030)
Occ. Tenure <sup>3</sup>	0.0002	0.0000	-0.0000	0.0001
	(0.0001)	(0.0001)	(0.0002)	(0.0001)
Old Job	-0.0342***	-0.0185**	-0.0285**	-0.0278**

	(0.0083)	(0.0079)	(0.0119)	(0.0121)
Female	-0.1130***	-0.1145***	-0.1192***	-0.1162***
	(0.0109)	(0.0115)	(0.0112)	(0.0125)
Hispanic	0.0262	0.0307	0.0334**	0.0463**
	(0.0218)	(0.0229)	(0.0166)	(0.0181)
Black	-0.0512***	-0.0568***	-0.0039	0.0059
	(0.0148)	(0.0158)	(0.0151)	(0.0167)
Constant	1.9733***	2.0127***	2.1188***	2.1442***
	(0.0619)	(0.0664)	(0.0931)	(0.1019)
N	24832	24832	18931	18931

All regressions include industry and occupation fixed effects.

Standard errors in parentheses are clustered at individual level.

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

	NLSY79 (1978–1995)		NLSY97 (1996–2014)	
	(1)	(2)	(3)	(4)
	OLS	IV-OLS	OLS	IV-OLS
Mismatch	-0.1153	-0.2246	-0.5853***	-0.5572***
	(0.1564)	(0.1615)	(0.1387)	(0.1452)
Worker Skill (Average)	0.1682*	0.3373***	0.1366**	0.0935
	(0.0955)	(0.1095)	(0.0696)	(0.0800)
Occ. Requirement (Average)	0.7047***	0.5877***	0.3468***	0.3793***
	(0.1008)	(0.1057)	(0.0822)	(0.0927)
Skill× Occ. Tenure	0.0250	-0.0268	0.0168	0.0374*
	(0.0195)	(0.0223)	(0.0210)	(0.0209)

## Table B4: Job Specialization with some College Education

Requirement $\times$ Occ. Tenure	-0.0049	0.0220	0.0664***	0.0535***
	(0.0137)	(0.0147)	(0.0191)	(0.0199)
Experience	-0.0109	0.0436***	0.0278***	0.0951***
	(0.0105)	(0.0142)	(0.0088)	(0.0120)
Experience <sup>2</sup>	0.0043***	-0.0003	0.0001	-0.0059***
	(0.0014)	(0.0017)	(0.0013)	(0.0016)
Experience <sup>3</sup>	-0.0002***	0.0000	-0.0000	0.0002***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Job Tenure	0.0387**	-0.0163	0.0054	-0.0201
	(0.0196)	(0.0220)	(0.0259)	(0.0234)
Job Tenure <sup>2</sup>	-0.0062*	0.0006	-0.0047	0.0003
	(0.0032)	(0.0035)	(0.0053)	(0.0041)
Job Tenure <sup>3</sup>	0.0003*	0.0000	0.0003	0.0001
	(0.0001)	(0.0002)	(0.0003)	(0.0002)
Occ. Tenure	0.0644***	0.0446*	0.0183	-0.0528*
	(0.0197)	(0.0233)	(0.0280)	(0.0283)
Occ. Tenure <sup>2</sup>	-0.0052*	-0.0035	0.0002	0.0028
	(0.0027)	(0.0033)	(0.0052)	(0.0043)
Occ. Tenure <sup>3</sup>	0.0001	0.0001	-0.0003	-0.0003
	(0.0001)	(0.0002)	(0.0003)	(0.0002)
Old Job	-0.0550***	-0.0390***	-0.0102	-0.0155
	(0.0143)	(0.0134)	(0.0134)	(0.0108)
Female	-0.1155***	-0.1071***	-0.1404***	-0.1427***
	(0.0186)	(0.0195)	(0.0158)	(0.0170)
Hispanic	0.0186	0.0250	0.0399*	0.0524**

	(0.0327)	(0.0352)	(0.0212)	(0.0233)
Black	-0.0563**	-0.0577**	-0.0106	-0.0087
	(0.0249)	(0.0266)	(0.0195)	(0.0210)
Constant	1.6902***	1.5894***	1.9045***	1.8853***
	(0.0928)	(0.1068)	(0.0983)	(0.1046)
N	10441	10441	11260	11260

53

All regressions include industry and occupation fixed effects.

Standard errors in parentheses are clustered at individual level.

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

## Table B5: Job Specialization with Higher than College Education

	NLSY79 (1978–1995)		NLSY97 (1996–2014)	
	(1)	(2)	(3)	(4)
	OLS	IV-OLS	OLS	IV-OLS
Mismatch	-0.4534**	-0.4734**	-0.3389*	-0.4486**
	(0.2281)	(0.2332)	(0.2003)	(0.2099)
Worker Skill (Average)	0.4295***	0.3501***	0.4449***	0.3271***
	(0.1347)	(0.1333)	(0.1032)	(0.1125)
Occ. Requirement (Average)	0.3339**	0.2065	0.7224***	0.6373***
	(0.1546)	(0.1661)	(0.1161)	(0.1340)
Skill $ imes$ Occ. Tenure	0.0393*	0.0471***	0.0190	0.0337
	(0.0213)	(0.0176)	(0.0251)	(0.0236)
Requirement × Occ. Tenure	0.0184	0.0478***	0.0201	0.0393*
	(0.0172)	(0.0171)	(0.0224)	(0.0207)
Experience	-0.0054	0.1389***	-0.0121	0.2028***
	(0.0152)	(0.0325)	(0.0123)	(0.0298)

Experience <sup>2</sup>	0.0021	-0.0099***	0.0039***	-0.0156***
	(0.0015)	(0.0029)	(0.0014)	(0.0031)
Experience <sup>3</sup>	-0.0001	0.0003***	-0.0002***	0.0004***
	(0.0000)	(0.0001)	(0.0001)	(0.0001)
Job Tenure	0.0124	-0.0328*	0.0290	-0.0299
	(0.0191)	(0.0175)	(0.0281)	(0.0257)
Job Tenure <sup>2</sup>	-0.0062**	0.0008	-0.0098*	-0.0004
	(0.0031)	(0.0027)	(0.0058)	(0.0048)
Job Tenure <sup>3</sup>	$0.0004^{***}$	0.0001	0.0005	0.0001
,	(0.0001)	(0.0001)	(0.0003)	(0.0003)
Occ. Tenure	0.0523**	0.0224	0.0282	-0.0336
	(0.0223)	(0.0221)	(0.0321)	(0.0334)
Occ. Tenure <sup>2</sup>	-0.0038*	-0.0061***	0.0035	0.0029
Occ. Tentile	(0.0022)	(0.0020)	(0.0055)	(0.0023)
Oce Tenurol	0.0000	0.0001*	0.0004	0.0002
Occ. Tenure <sup>3</sup>	-0.0000 (0.0001)	0.0001* (0.0001)	-0.0004 (0.0003)	-0.0002 (0.0003)
Old Job	0.0268*	-0.0067	-0.0035	-0.0280**
	(0.0158)	(0.0140)	(0.0160)	(0.0120)
Female	-0.1206***	-0.1172***	-0.1054***	-0.1282***
	(0.0203)	(0.0210)	(0.0190)	(0.0206)
Hispanic	0.0505	0.0513	0.0481	0.0595*
	(0.0543)	(0.0572)	(0.0309)	(0.0332)
Black	-0.0579	-0.0468	-0.0333	-0.0261
	(0.0430)	(0.0446)	(0.0281)	(0.0293)
Constant	1.9106***	1.6561***	2.1281***	1.7753***

MURALI	M	UI	RA	L	
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	(0.1922)	(0.2164)	(0.1650)	(0.1898)
Ν	9613	9613	11673	11673

All regressions include industry and occupation fixed effects.

Standard errors in parentheses are clustered at individual level.

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

### **B.2** Labor Market Turnover across Education

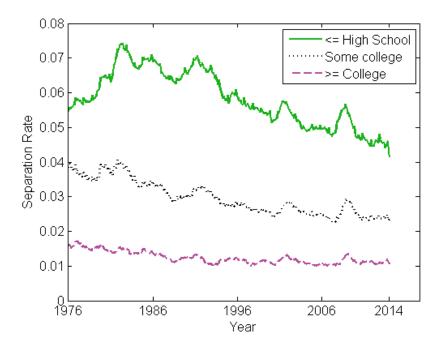


Figure B1: Separation rate across education

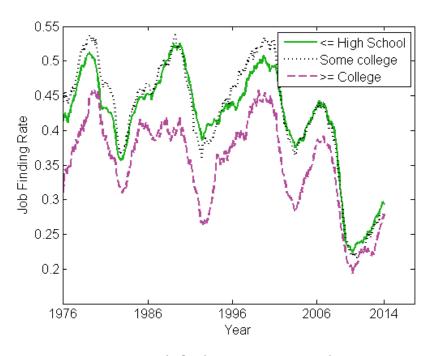


Figure B2: Job finding rate across education

# C. Model

# C.1 Proof of Proposition 1

Assumption: There is free entry of vacancies. In equilibrium,  $V(f) = 0 \ \forall f \in [0, 2\pi]$ We need to prove  $u(w) = u \Leftrightarrow v(f) = v \ \forall w, f \in [0, 2\pi]$ .

$$(\Rightarrow) \ u(w) = u \Rightarrow v(f) = v \ \forall f, w \in [0, 2\pi].$$

*Proof.* Suppose v(f') > v(f) for some f and f'. This implies  $\theta(f') > \theta(f)$ , and hence q(f') < q(f). Thus, V(f') < V(f), which violates the free entry condition.

 $(\Leftarrow) \ v(f) = v \Rightarrow u(w) = u \ \forall f, w \in [0, 2\pi].$ 

*Proof.* Suppose u(w') > u(w) for some w and w'. This implies  $\theta(w') < \theta(w)$ , and hence q(w') > q(w). Thus, it is profitable for the firm at w to deviate and create a vacancy at w' as V(w') > V(w), violating the free entry condition.

Thus, under free entry of vacancies,  $u(w) = u \Leftrightarrow v(f) = v \; \forall w, f \in [0, 2\pi]$ 

## C.2 Derivation of Equilibrium Conditions

The equilibrium conditions consists of free entry condition, definition of cutoff mismatch, and the definition of cutoff productivity.

Free entry condition gives

$$c = \frac{2q(\theta)(1-\beta)}{r+\lambda} \left[ \int_0^{\bar{x}} \eta(x)\bar{\epsilon}dx - b\bar{x} - \frac{\beta c\theta\bar{x}}{1-\beta} \right] + \frac{2q(\theta)\lambda}{r+\lambda} \int_0^{\bar{x}} \int_{\epsilon^*(\tau)}^{\bar{\epsilon}} J(\tau,z)dF(z)d\tau.$$
(36)

The definition of cutoff mismatch is

$$(1-\beta)\left[\eta(\bar{x})\bar{\epsilon}-b\right] - \beta c\theta + \lambda \int_{\epsilon^*(\bar{x})}^{\bar{\epsilon}} J(\bar{x},z)dF(z) = 0.$$
(37)

Cutoff productivity satisfies

$$(1-\beta)\left[\eta(x)\epsilon^*(x)-b\right] - \beta c\theta + \lambda \int_{\epsilon^*(x)}^{\bar{\epsilon}} J(x,z)dF(z) = 0.$$
(38)

We can combine the definition of  $J(x, \epsilon)$  with the definition of cutoff productivity to get a simplified solution for *J* in terms of cutoff productivity as

$$J(x,\epsilon) = \frac{(1-\beta)\eta(x)}{r+\lambda} \left[\epsilon - \epsilon^*(x)\right].$$
(39)

Plugging this equation for *J* back into the original equilibrium equations gives us the final equilibrium conditions to solve for  $\theta$ ,  $\bar{x}$  and  $\epsilon^*(x)$ .